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Two Summation Formulae Relating Hypergeometric Function

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Two Summation Formulae Relating Hypergeometric Function

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Abstract - The aim of the present paper is to obtain two summation formulae associated to Hypergeometric function.The results derived in this paper are of general character and are believed to be new.

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I. Introduction

The special function is one of the central branches of Mathematical sciences initiated by LEuler .But systematic study of the Hypergeometric functions were initiated by C.F Gauss, an imminent German Mathematician in 1812 by defining the Hypergeometric series and he had also proposed notation for Hypergeometric functions. Since about 250 years several talented brains and promising Scholars have been contributed to this area. Some of them are C.F Gauss, G.H Hardy , S. Ramanujan ,A.P Prudnikov , W.W Bell , Yu. A Brychkov and G.E Andrews.

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ & \\ b_1, b_2, \cdots, b_B & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{ll} (a_A) & ; \\ & \\ (b_B) & ; \end{array} z\right] \equiv {}_AF_B\left[\begin{array}{ll} (a_j)_{j=1}^{A} & ; \\ & \\ (b_j)_{j=1}^{B} & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \qquad (1) $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers. The series converges for all finite z if $A \leq B$, converges for$|\ z\ | < 1$ if A=B+1, diverges for all z, $z \neq 0$ if$A > B + 1$.

Contiguous Relation is defined by

Following Eq. (10), p-51 of ref [6], we write

$$ (a-b)\ {}_2F_1\left[\begin{array}{ll} a,\ b\ ; \\ c\ ; \end{array} z\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1,\ b\ ; \\ c\ ; \end{array} z\right] - b\ {}_2F_1\left[\begin{array}{ll} a,\ b+1\ ; \\ c\ ; \end{array} z\right] \qquad (2) $$

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Ref.
6. Rainville, E. D.; The contiguous function relations for ${}_pF_q$ with applications to Bateman's $J_n^{u,v}$ and Rice's $H_n\ (\zeta, p, \nu)$, *Bull. Amer. Math. Soc.*, 51(1945), 714-723.

Recurrence relation is defined by

$$\Gamma(z+1) = z\,\Gamma(z) \tag{3}$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.3)]

$${}_2F_1\left[\begin{array}{c} a,\ b\ ; \\ \frac{a+b+1}{2}\ ; \end{array}\ \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\,\Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\,\Gamma(\frac{b+1}{2})} \tag{4}$$

$$= \frac{2^{(b-1)}\,\Gamma(\frac{b}{2})\,\Gamma(\frac{a+b+1}{2})}{\Gamma(b)\,\Gamma(\frac{a+1}{2})} \tag{5}$$

II. Main Results of Summation Formulae

$${}_2F_1\left[\begin{array}{c} a,\ b\ ; \\ \frac{a+b+23}{2}\ ; \end{array}\ \frac{1}{2}\right] = \frac{2^b\,\Gamma(\frac{a+b+23}{2})}{(a-b)\,\Gamma(b)}\times$$

$$\times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{1024a(654729075-1396704420a+1094071221a^2-444647600a^3+107494190a^4)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+\right.\right.$$

$$+\frac{1024a(-16486680a^5+1646778a^6-106800a^7+4335a^8-100a^9+a^{10}-400914000b)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(4564470450ab-1410623712a^2b+1263684888a^3b-155769600a^4b+42918540a^5b)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(-2331168a^6b+255192a^7b-5040a^8b+210a^9b+2644887945b^2-265793584ab^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(3183848164a^2b^2-293010704a^3b^2+257688830a^4b^2-11918928a^5b^2+3222324a^6b^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(-57456a^7b^2+5985a^8b^2+368444608b^3+2290676024ab^3-33209568a^2b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(529562376a^3b^3-17364480a^4b^3+14271432a^5b^3-217056a^6b^3+54264a^7b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(407004318b^4+126838376ab^4+413414806a^2b^4-904400a^3b^4+26340650a^4b^4)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(-271320a^5b^4+203490a^6b^4+32111520b^5+117320364ab^5+9767520a^2b^5)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(21434280a^3b^5+352716a^5b^5+9231474b^6+4019792ab^6+7533652a^2b^6+180880a^3b^6)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(293930a^4b^6+357312b^7+1020984ab^7+93024a^2b^7+116280a^3b^7+38367b^8+14364ab^8)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024a(20349a^2b^8+560b^9+1330ab^9+21b^{10})}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024b(654729075-400914000a+2644887945a^2+368444608a^3+407004318a^4+32111520a^5)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(9231474a^6+357312a^7+38367a^8+560a^9+21a^{10}-1396704420b+4564470450ab)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(-265793584a^2b+2290676024a^3b+126838376a^4b+117320364a^5b+4019792a^6b)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(1020984a^7b+14364a^8b+1330a^9b+1094071221b^2-1410623712ab^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(3183848164a^2b^2-33209568a^3b^2+413414806a^4b^2+9767520a^5b^2+7533652a^6b^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(93024a^7b^2+20349a^8b^2-444647600b^3+1263684888ab^3-293010704a^2b^3)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(529562376a^3b^3-904400a^4b^3+21434280a^5b^3+180880a^6b^3+116280a^7b^3)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(107494190b^4-155769600ab^4+257688830a^2b^4-17364480a^3b^4+26340650a^4b^4)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(293930a^6b^4-16486680b^5+42918540ab^5-11918928a^2b^5+14271432a^3b^5)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

Notes

$$+\frac{1024b(-271320a^4b^5+352716a^5b^5+1646778b^6-2331168ab^6+3222324a^2b^6)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{(1024b-217056a^3b^6+203490a^4b^6-106800b^7+255192ab^7-57456a^2b^7+54264a^3b^7)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024b(4335b^8-5040ab^8+5985a^2b^8-100b^9+210ab^9+b^{10})}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{2048(654729075+400914000a+2644887945a^2-368444608a^3+407004318a^4}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(-32111520a^5+9231474a^6-357312a^7+38367a^8-560a^9+21a^{10}+1396704420b)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(4564470450ab+265793584a^2b+2290676024a^3b-126838376a^4b+117320364a^5b)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(-4019792a^6b+1020984a^7b-14364a^8b+1330a^9b+1094071221b^2+1410623712ab^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(3183848164a^2b^2+33209568a^3b^2+413414806a^4b^2-9767520a^5b^2+7533652a^6b^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(-93024a^7b^2+20349a^8b^2+444647600b^3+1263684888ab^3+293010704a^2b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(529562376a^3b^3+904400a^4b^3+21434280a^5b^3-180880a^6b^3+116280a^7b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(107494190b^4+155769600ab^4+257688830a^2b^4+17364480a^3b^4+26340650a^4b^4)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(293930a^6b^4+16486680b^5+42918540ab^5+11918928a^2b^5+14271432a^3b^5+271320a^4b^5)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

Notes

$$+\frac{2048(352716a^5b^5+1646778b^6+2331168ab^6+3222324a^2b^6+217056a^3b^6+203490a^4b^6)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(+106800b^7+255192ab^7+57456a^2b^7+54264a^3b^7+4335b^8+5040ab^8+5985a^2b^8)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(100b^9+210ab^9+b^{10})}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{2048(654729075+1396704420a+1094071221a^2+444647600a^3+107494190a^4+16486680a^5)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(1646778a^6+106800a^7+4335a^8+100a^9+a^{10}+400914000b+4564470450ab)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(1410623712a^2b+1263684888a^3b+155769600a^4b+42918540a^5b+2331168a^6b)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(255192a^7b+5040a^8b+210a^9b+2644887945b^2+265793584ab^2+3183848164a^2b^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(293010704a^3b^2+257688830a^4b^2+11918928a^5b^2+3222324a^6b^2+57456a^7b^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(5985a^8b^2-368444608b^3+2290676024ab^3+33209568a^2b^3+529562376a^3b^3)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(17364480a^4b^3+14271432a^5b^3+217056a^6b^3+54264a^7b^3+407004318b^4)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(-126838376ab^4+413414806a^2b^4+904400a^3b^4+26340650a^4b^4+271320a^5b^4)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(203490a^6b^4-32111520b^5+117320364ab^5-9767520a^2b^5+21434280a^3b^5)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(352716a^5b^5+9231474b^6-4019792ab^6+7533652a^2b^6-180880a^3b^6+293930a^4b^6)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

Notes

$$+\frac{2048(-357312b^7+1020984ab^7-93024a^2b^7+116280a^3b^7+38367b^8-14364ab^8)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{2048(20349a^2b^8-560b^9+1330ab^9+21b^{10})}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}\Bigg\}\Bigg] \tag{6}$$

$${}_2F_1\left[\begin{matrix}a,\ b\ ;\\ \frac{a+b+24}{2}\ ;\end{matrix}\ \frac{1}{2}\right]=\frac{2^b\ \Gamma(\frac{a+b+24}{2})}{(a-b)\ \Gamma(b)}\times$$

$$\times\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{2048(3715891200a-5441863680a^2+3264915456a^3-1076416000a^4)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(218683520a^5-28865760a^6+2524368a^7-145200a^8+5280a^9-110a^{10}+a^{11})}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(3715891200b+18690693120a^2b-4089046016a^3b+3093104256a^4b-317412480a^5b)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(75431664a^6b-3589344a^7b+347424a^8b-6160a^9b+231a^{10}b+5441863680b^2)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(18690693120ab^2+9866191104a^3b^2-699103328a^4b^2+531899984a^5b^2-21114016a^6b^2)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(4975872a^7b^2-79002a^8b^2+7315a^9b^2+3264915456b^3+4089046016ab^3)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(9866191104a^2b^3+1327912432a^4b^3-35814240a^5b^3+25467904a^6b^3-341088a^7b^3)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(74613a^8b^3+1076416000b^4+3093104256ab^4+699103328a^2b^4+1327912432a^3b^4)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(55711040a^5b^4-497420a^6b^4+319770a^7b^4+218683520b^5+317412480ab^5)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(531899984a^2b^5+35814240a^3b^5+55711040a^4b^5+646646a^6b^5+28865760b^6)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(75431664ab^6+21114016a^2b^6+25467904a^3b^6+497420a^4b^6+646646a^5b^6+2524368b^7)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(3589344ab^7+4975872a^2b^7+341088a^3b^7+319770a^4b^7+145200b^8+347424ab^8)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(79002a^2b^8+74613a^3b^8+5280b^9+6160ab^9+7315a^2b^9+110b^{10}+231ab^{10}+b^{11})}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096b(3715891200+1199554560a+4962674688a^2+720247296a^3+469992064a^4)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(34181280a^5+7691376a^6+270864a^7+24816a^8+330a^9+11a^{10}-1199554560b)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(12030259200ab+1008349696a^2b+3230041600a^3b+198001888a^4b+113212512a^5b)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(3702160a^6b+743424a^7b+9702a^8b+770a^9b+4962674688b^2-1008349696ab^2)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(5777911552a^2b^2+181722688a^3b^2+473992848a^4b^2+12633936a^5b^2+6273344a^6b^2)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(75240a^7b^2+13167a^8b^2-720247296b^3+3230041600ab^3-181722688a^2b^3)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(747974976a^3b^3+9586640a^4b^3+20837376a^5b^3+198968a^6b^3+85272a^7b^3)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(469992064b^4-198001888ab^4+473992848a^2b^4-9586640a^3b^4+30749600a^4b^4)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(135660a^5b^4+248710a^6b^4-34181280b^5+113212512ab^5-12633936a^2b^5)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(20837376a^3b^5-135660a^4b^5+352716a^5b^5+7691376b^6-3702160ab^6)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

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$$+\frac{4096b(6273344a^2b^6-198968a^3b^6+248710a^4b^6-270864b^7+743424ab^7-75240a^2b^7)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{4096b(+85272a^3b^7+24816b^8-9702ab^8+13167a^2b^8-330b^9+770ab^9+11b^10)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}\Big\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Big\{\frac{4096a(3715891200-1199554560a+4962674688a^2-720247296a^3+469992064a^4)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(-34181280a^5+7691376a^6-270864a^7+24816a^8-330a^9+11a^{10}+1199554560b)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(12030259200ab-1008349696a^2b+3230041600a^3b-198001888a^4b+113212512a^5b)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(-3702160a^6b+743424a^7b-9702a^8b+770a^9b+4962674688b^2+1008349696ab^2)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(5777911552a^2b^2-181722688a^3b^2+473992848a^4b^2-12633936a^5b^2+6273344a^6b^2)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(-75240a^7b^2+13167a^8b^2+720247296b^3+3230041600ab^3+181722688a^2b^3)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(747974976a^3b^3-9586640a^4b^3+20837376a^5b^3-198968a^6b^3+85272a^7b^3)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(469992064b^4+198001888ab^4+473992848a^2b^4+9586640a^3b^4+30749600a^4b^4)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(-135660a^5b^4+248710a^6b^4+34181280b^5+113212512ab^5+12633936a^2b^5)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(20837376a^3b^5+135660a^4b^5+352716a^5b^5+7691376b^6+3702160ab^6+6273344a^2b^6)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{4096a(198968a^3b^6+248710a^4b^6+270864b^7+743424ab^7+75240a^2b^7+85272a^3b^7)}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

Notes

$$+\frac{4096a(24816b^8+9702ab^8+13167a^2b^8+330b^9+770ab^9+11b^{10})}{\Big[\prod_{\eta=0}^{10}\{a-b-2\eta\}\Big]\Big[\prod_{\vartheta=1}^{11}\{a-b+2\vartheta\}\Big]}+$$

$$+\frac{2048(3715891200a+5441863680a^2+3264915456a^3+1076416000a^4+218683520a^5)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

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$$+\frac{2048(28865760a^6+2524368a^7+145200a^8+5280a^9+110a^{10}+a^{11}+3715891200b)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(18690693120a^2b+4089046016a^3b+3093104256a^4b+317412480a^5b+75431664a^6b)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(3589344a^7b+347424a^8b+6160a^9b+231a^10b-5441863680b^2+18690693120ab^2)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(9866191104a^3b^2+699103328a^4b^2+531899984a^5b^2+21114016a^6b^2+4975872a^7b^2)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(79002a^8b^2+7315a^9b^2+3264915456b^3-4089046016ab^3+9866191104a^2b^3)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(1327912432a^4b^3+35814240a^5b^3+25467904a^6b^3+341088a^7b^3+74613a^8b^3)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(-1076416000b^4+3093104256ab^4-699103328a^2b^4+1327912432a^3b^4+55711040a^5b^4)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(497420a^6b^4+319770a^7b^4+218683520b^5-317412480ab^5+531899984a^2b^5)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(-35814240a^3b^5+55711040a^4b^5+646646a^6b^5-28865760b^6+75431664ab^6)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(-21114016a^2b^6+25467904a^3b^6-497420a^4b^6+646646a^5b^6+2524368b^7-3589344ab^7)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(4975872a^2b^7-341088a^3b^7+319770a^4b^7-145200b^8+347424ab^8)}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}+$$

$$+\frac{2048(-79002a^2b^8+74613a^3b^8+5280b^9-6160ab^9+7315a^2b^9-110b^{10}+231ab^{10}+b^{11})}{\Big[\prod_{\delta=0}^{11}\{a-b-2\delta\}\Big]\Big[\prod_{\zeta=1}^{10}\{a-b+2\zeta\}\Big]}\Bigg\}\Bigg] \tag{7}$$

III. Derivation of Summation Formula (6)

Substituting $c=\frac{a+b+23}{2}$ and $z=\frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix}a,\ b\\ \frac{a+b+23}{2}\end{matrix};\ \frac{1}{2}\right]=a\ {}_2F_1\left[\begin{matrix}a+1,\ b\\ \frac{a+b+23}{2}\end{matrix};\ \frac{1}{2}\right]-b\ {}_2F_1\left[\begin{matrix}a,\ b+1\\ \frac{a+b+23}{2}\end{matrix};\ \frac{1}{2}\right]$$

Now applying the formula obtained by Salahuddin [Salahuddin.,p.12(9)], we get

$$L.H.S=a\ \frac{2^b\ \Gamma(\frac{a+b+23}{2})}{\Gamma(b)}\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{1024(654729075-1396704420a+1094071221a^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(-444647600a^3+107494190a^4-16486680a^5+1646778a^6-106800a^7+4335a^8-100a^9)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(a^{10}-400914000b+4564470450ab-1410623712a^2b+1263684888a^3b-155769600a^4b)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(42918540a^5b-2331168a^6b+255192a^7b-5040a^8b+210a^9b+26448879 45b^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(-265793584ab^2+3183848164a^2b^2-293010704a^3b^2+257688830a^4b^2-11918928a^5b^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(3222324a^6b^2-57456a^7b^2+5985a^8b^2+368444608b^3+2290676024ab^3-33209568a^2b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(529562376a^3b^3-17364480a^4b^3+14271432a^5b^3-217056a^6b^3+54264a^7b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(407004318b^4+126838376ab^4+413414806a^2b^4-904400a^3b^4+26340650a^4b^4)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(-271320a^5b^4+203490a^6b^4+32111520b^5+117320364ab^5+9767520a^2b^5)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(21434280a^3b^5+352716a^5b^5+9231474b^6+4019792ab^6+7533652a^2b^6+180880a^3b^6)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(293930a^4b^6+357312b^7+1020984ab^7+93024a^2b^7+116280a^3b^7)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(38367b^8+14364ab^8+20349a^2b^8+560b^9+1330ab^9+21b^{10})}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}\Big\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+2}{2})}\Big\{\frac{1024(654729075+400914000a+2644887945a^2-368444608a^3+407004318a^4)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(-32111520a^5+9231474a^6-357312a^7+38367a^8-560a^9+21a^{10}+1396704420b)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(4564470450ab+265793584a^2b+2290676024a^3b-126838376a^4b+117320364a^5b)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(-4019792a^6b+1020984a^7b-14364a^8b+1330a^9b+1094071221b^2+1410623712ab^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(3183848164a^2b^2+33209568a^3b^2+413414806a^4b^2-9767520a^5b^2+7533652a^6b^2)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(-93024a^7b^2+20349a^8b^2+444647600b^3+1263684888ab^3+293010704a^2b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(529562376a^3b^3+904400a^4b^3+21434280a^5b^3-180880a^6b^3+116280a^7b^3)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(107494190b^4+155769600ab^4+257688830a^2b^4+17364480a^3b^4+26340650a^4b^4)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(293930a^6b^4+16486680b^5+42918540ab^5+11918928a^2b^5+14271432a^3b^5+271320a^4b^5)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

Notes

$$+\frac{1024(352716a^5b^5+1646778b^6+2331168ab^6+3222324a^2b^6+217056a^3b^6+203490a^4b^6)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(106800b^7+255192ab^7+57456a^2b^7+54264a^3b^7+4335b^8+5040ab^8+5985a^2b^8)}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}+$$

$$+\frac{1024(100b^9+210ab^9+b^{10})}{\Big[\prod_{\varphi=1}^{10}\{a-b-(2\varphi-1)\}\Big]\Big[\prod_{\omega=1}^{11}\{a-b+(2\omega-1)\}\Big]}\Bigg\}\Bigg]-$$

$$-b\,\frac{2^{b+1}\,\Gamma(\frac{a+b+23}{2})}{\Gamma(b+1)}\Bigg[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{1024(654729075+1396704420a+1094071221a^2+444647600a^3)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(107494190a^4+16486680a^5+1646778a^6+106800a^7+4335a^8+100a^9+a^{10})}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(400914000b+4564470450ab+1410623712a^2b+1263684888a^3b+155769600a^4b)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(42918540a^5b+2331168a^6b+255192a^7b+5040a^8b+210a^9b+2644887945b^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(265793584ab^2+3183848164a^2b^2+293010704a^3b^2+257688830a^4b^2+11918928a^5b^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(3222324a^6b^2+57456a^7b^2+5985a^8b^2-368444608b^3+2290676024ab^3+33209568a^2b^3)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(529562376a^3b^3+17364480a^4b^3+14271432a^5b^3+217056a^6b^3+54264a^7b^3)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(407004318b^4-126838376ab^4+413414806a^2b^4+904400a^3b^4+26340650a^4b^4)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(271320a^5b^4+203490a^6b^4-32111520b^5+117320364ab^5-9767520a^2b^5+21434280a^3b^5)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(352716a^5b^5+9231474b^6-4019792ab^6+7533652a^2b^6-180880a^3b^6+293930a^4b^6)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

Notes

$$+\frac{1024(-357312b^7+1020984ab^7-93024a^2b^7+116280a^3b^7+38367b^8-14364ab^8+20349a^2b^8)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(-560b^9+1330ab^9+21b^{10})}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}\Bigg\}-$$

Notes

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{1024(654729075-400914000a+2644887945a^2+368444608a^3+407004318a^4)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(32111520a^5+9231474a^6+357312a^7+38367a^8+560a^9+21a^{10}-1396704420b)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(4564470450ab-265793584a^2b+2290676024a^3b+126838376a^4b+117320364a^5b)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(4019792a^6b+1020984a^7b+14364a^8b+1330a^9b+1094071221b^2-1410623712ab^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(3183848164a^2b^2-33209568a^3b^2+413414806a^4b^2+9767520a^5b^2+7533652a^6b^2)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(93024a^7b^2+20349a^8b^2-444647600b^3+1263684888ab^3-293010704a^2b^3)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(529562376a^3b^3-904400a^4b^3+21434280a^5b^3+180880a^6b^3+116280a^7b^3+107494190b^4)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(-155769600ab^4+257688830a^2b^4-17364480a^3b^4+26340650a^4b^4+293930a^6b^4)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(-16486680b^5+42918540ab^5-11918928a^2b^5+14271432a^3b^5-271320a^4b^5)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(352716a^5b^5+1646778b^6-2331168ab^6+3222324a^2b^6-217056a^3b^6+203490a^4b^6)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(-106800b^7+255192ab^7-57456a^2b^7+54264a^3b^7+4335b^8-5040ab^8+5985a^2b^8)}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}+$$

$$+\frac{1024(-100b^9+210ab^9+b^{10})}{\Big[\prod_{\varsigma=1}^{11}\{a-b-(2\varsigma-1)\}\Big]\Big[\prod_{\tau=1}^{10}\{a-b+(2\tau-1)\}\Big]}\Big\}\Big]$$

On simplification ,we get the result (6).

On the same way, we can prove the result (7).

IV. Conclusion

In this paper we have derived two summation formulae with the help of contiguous relation . However, the formulae presented herein may be further developed to extend this result .Thus we can only hope that the development presented in this work will stimulate further interest and research in this important area of classical special functions. Just as the mathematical properties of the Gauss hypergeometric function are already of immense and significant utility in mathematical sciences and numerous other areas of pure and applied mathematics, the elucidation and discovery of the formulae of hypergeometric functions considered herein should certainly eventually prove useful to further developments in the broad areas alluded to above.

References Références Referencias

1. Abramowitz, Milton., A and Stegun, Irene , *Handbook of Mathematical Functions with Formulas , Graphs , and Mathematical Tables*(National Bureau of Standards, 1970).
2. Arora, Asish, Singh, Rahul , Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems *Journal of Rajasthan Academy of Physical Sciences.*, 7(2008), 335-342.
3. Bells, Richard. Wong, Roderick; *Special Functions, A Graduate Text*(Cambridge Studies in Advanced Mathematics, 2010).
4. Luke, Y.L, *Mathematical Functions and their Approximations*(Academic Press Inc, 1975).
5. Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; *Integrals and Series Vol. 3: More Special Functions.* Nauka, Moscow, 1986. Translated from the Russian by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne, 1990.
6. Rainville, E. D.; The contiguous function relations for ${}_pF_q$ with applications to Bateman's $J_n^{u,v}$ and Rice's $H_n\ (\zeta,p,\nu)$, *Bull. Amer. Math. Soc.*, 51(1945), 714-723.
7. Salahuddin ; Two Summation Formulae Based On Half Argument Associated To Hypergeometric Function, *Global journal of Science Frontier Research*, 10(2010),08-35.

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